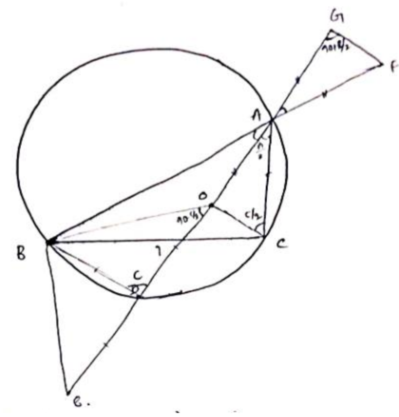
01.03.2024 - Cash Award Math Rider I Prize Winner Ms.Naga Bhavya Jothi's Solution



Construction:

Join OB, OC, BD

Proof:

O → incenter of ΔABC.

As AO = AG and AC = AF and $\angle OAC = \angle GAF = \frac{A}{2}$

$\Delta AOC \cong \Delta AGF$

$\Longrightarrow \angle AGF = 90^{\circ\circ} + \frac{\angle B}{2}$ ----------------- (1) (as $\angle AOC = 180° - \frac{A}{2} - \frac{C}{2} = 90° + \frac{B}{2} = \angle AGF$)

Now, $\angle BOE = \angle ABO + \angle BAO = \frac{\angle A}{2} + \frac{\angle B}{2} = 90° - \frac{C}{2}$

And for $\overline{AB}$; $\angle ADB = \angle ACB = \angle C$ (as D lie on circumcircle)

$\Longrightarrow$ In ΔBDO; $\angle DBO = 90° - \frac{\angle C}{2}$ $\Longrightarrow BD = OD$

$\angle BOD = 90° - \frac{\angle C}{2}$

$\angle BOD = \angle C$

In $\Delta BEO; OD = BD = DE$ $\Longrightarrow \angle OBE = 90°$

$\Longrightarrow \angle ABE = \angle ABO + \angle OBE = 90° + \frac{\angle B}{2} = \angle FBE$ ---------------- (2)

From (1) & (2)

$\Longrightarrow$ ∠AGF = ∠EGF = ∠FBE

$\Longrightarrow$ EBGF is a cyclic Quadrilateral (i.e .,BEFG is concyclic)

--------------------------- Hence Proved